A coupled Domain Decomposition - Model Order Reduction framework for the Simulation of Domain Growth Problems: Application to 3D Printing

General Background – Industrial Issues

3D Printing, also termed **Additive Manufacturing** (**AM**) is revolutionizing the manufacturing world. It allows for building parts of unprecedented geometrical complexity, comprising internal cooling channels and lattice structures (Figure 1). Furthermore, AM processes minimize the use of raw materials compared with conventional production methods and enable the repair of parts rather than their replacement. They therefore contribute to a more sustainable and resource-efficient industry.

Figure 1. The 3D printed copper Aerospike rocket engine [Source: Fabbaloo]

However, **the successful 3D printing of metal parts remains a real challenge**. Indeed, thermal history, thermal gradients and cooling rates inherent to the process may induce **defects** such as residual stresses, geometric distortions, porosity or undesired microstructures (Figure 2 right), among many others. In addition, the printing process may be aborted due to the occurrence of instabilities (Figure 2 left). These issues can significantly affect the mechanical properties, dimensional accuracy, and overall reliability of the manufactured components.

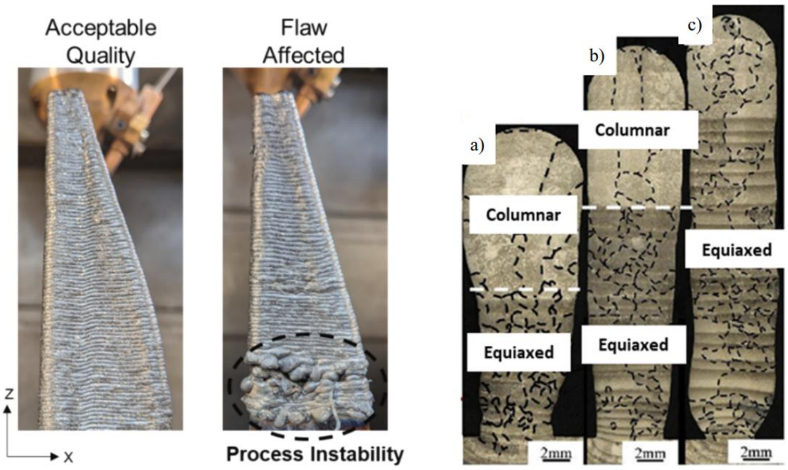

Figure 2 Left: process instabilities caused by inadequate thermal management [1], right: influence of the material deposition strategy on the proportion of columnar microstructures of a titanium alloy [2].

These defects are explained by the **thermal history** of the part: knowledge of the temperature field over time is a fundamental input for controlling the printing process and for understanding — and thus optimizing — the mechanical properties of the component. The so-called **solidification cooling**

rate and **solid cooling rate** are key quantities, responsible for numerous microstructural defects of the manufactured part (Figure 3).

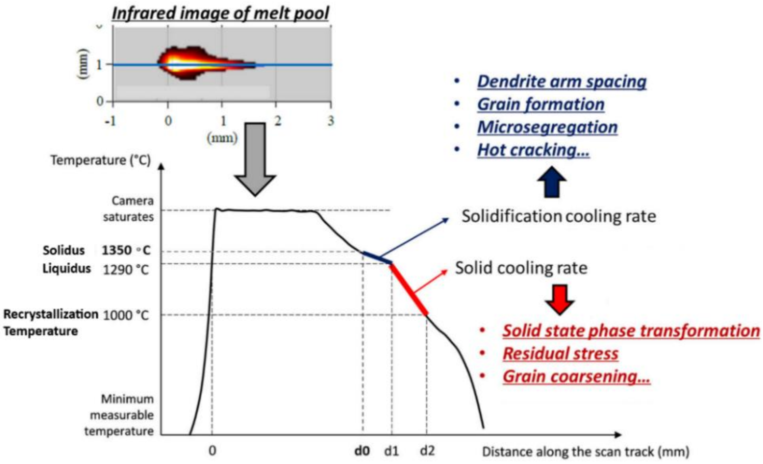

Figure 3. Relationship between cooling rates and microstructural/mechanical features [3].

It is in this context that numerical simulation of additive manufacturing processes has a major role to play: it should enable the **optimization of machine parameters** - e.g. **material deposition strategy** and **input power over time** - used during 3D printing while reducing the number of trials required to produce the first compliant part. High-fidelity simulations including thermal, fluid dynamics and vaporization aspects are, in principle, a possible solution for optimizing part production. However, **the associated computation times make them impractical at the scale of a full component.** Considering only thermal aspects greatly reduces computation times while preserving thermal fields and melt pool sizes consistent with experiments.

Nevertheless, **realistic thermal simulations of additive manufacturing processes remain orders of magnitudes too slow to develop a digital twin geared towards the optimization of machine parameters**. It is therefore essential to develop a radically innovative simulation strategy that **speeds up realistic – i.e. scan path resolved and fully nonlinear - thermal computations by orders of magnitudes**.

The high computational cost of realistic AM thermal simulations is caused by strong nonlinearities: **phase changes** (liquid/gas, solid/liquid and solid/solid), **temperature-dependent physical properties** and **convective/radiative heat exchanges**. Particularly cumbersome is the mandatory use of "small" time steps: **the time step size (Δt) must not exceed the ratio of the heat source radius ($r_{heat\ source}$) to its speed ($v_{heat\ source}$)**:

$$\Delta t < \frac{r_{heat\ source}}{v_{heat\ source}} \tag{1}$$

Indeed, severe accuracy loss in the vicinity of the melt pool occurs if the above relation is violated (Figure 4) independently of the chosen time integration scheme, and computed key quantities such as solidification rates become unreliable. Consequently, **millions of time steps are often needed to simulate the manufacturing of a realistic industrial part**.

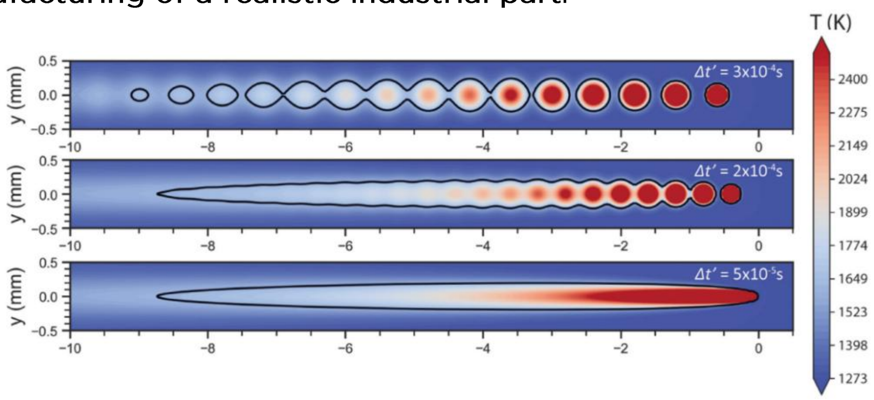

Figure 4. Increase of time step size leading to unacceptable accuracy loss in the vicinity of the melt pool [4].

Furthermore, **the computational domain constantly grows over time** (cf. Figure 5), which contributes to the high computational costs and requires a dedicated numerical implementation.

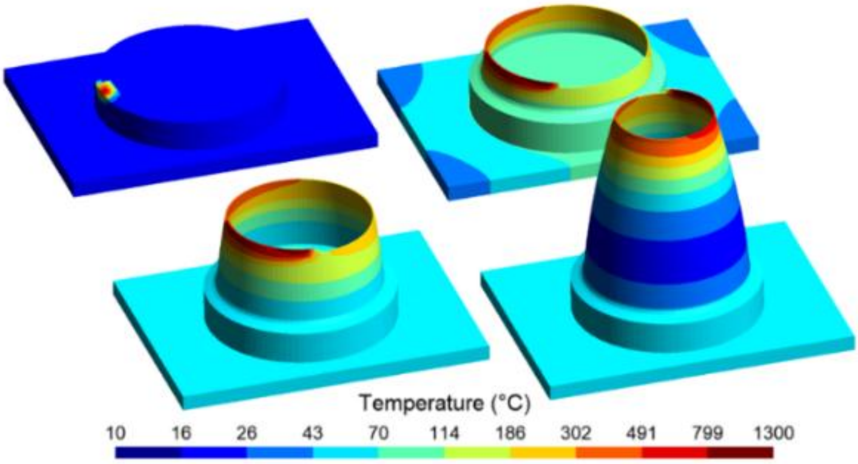

Figure 5 Temperature fields computed during the Additive Manufacturing of a nozzle

Reducing the computational time of DED simulations **by several orders of magnitude** is essential. Without this, running **multi-query simulations with varying input power and material deposition strategy** to optimize machine parameters would remain out of reach.

Envisioned Strategy

We have made significant contributions to the acceleration of **AM thermal computations in a Finite Element framework** [5], [6], [7], [8], notably through the combination of **Stabilized Explicit Scheme** with **Overlapping Schwarz Waveform Relaxation** for space-time parallelization.

The present work seeks to add the benefits of **Projection-based Reduced Order Models** (ROM) to our current framework. **Projection-based Reduced Order Models** (ROM) approximate the solution of a high-dimensional system by projecting its governing equations onto a low-dimensional subspace, typically constructed from a set of precomputed solution snapshots (e.g., via Proper Orthogonal Decomposition). In the linear case, this projection reduces the original system to a small linear system of equations, whose size is independent of the full-order model's dimension, yielding substantial computational savings while retaining a controllable approximation error. Such linear ROMs are particularly attractive when the underlying physics is well captured by a fixed reduced basis across the parameter or time range of interest, as is often the case for problems without strong nonlinearities or topology changes.

As is well known, linear ROM strategies are not able to efficiently reduce problems involving **moving heat sources** and **evolving domains** - characteristic of AM process simulations (cf. Figure 5) - due to the slow decay of the Kolmogorov *n*-width. This limitation underscores the need for **nonlinear ROM strategies**, which are inherently better equipped to capture such dynamics.

The last decade has witnessed a surge in the development of **nonlinear ROM.** In particular, **nonlinear ROM based on coordinate transformation** (**registration**) have proven effective for handling parametric systems with sharp parameter-dependent coherent structures with compact support such as shocks, shear layers, and cracks. Following the work of Taddei [9], registration-based methods have been extensively applied to inviscid and viscous shock-dominated flows in complex geometries [10], [11], [12] --- see also [13] for a related effort in fracture mechanics. Despite their success for steady problems, their impact for unsteady problems is limited due to difficulties in hyper-reduction and the cumbersome implementation [14]. In this project, we plan to use **registration methods** with **domain decomposition techniques** to devise a component-based ROM with a time-dependent domain partition. Registration techniques will be introduced to dynamically adjust the size and location of the approximation domain in the vicinity of the heat source. Domain decomposition will then be leveraged to couple near-field and far-field approximations in a consistent and efficient manner. We shall also resort to localized training [15] strategies to lower training costs and ultimately generate highly accurate local approximations without the need for full-order coupled solves. Furthermore, as in the present setting the coordinate transformation is driven by the source location --- which is known a priori --- we expect that our nonlinear approximation will generalize well outside the training regime. If successful, **this method would constitute the first application of registration techniques in combination with domain decomposition for solving unsteady PDEs.**

The project is also open to other ROM strategies, the main goal being efficiency in realistic AM simulations.

Applications, Supervisions, Collaborations

The present job offer targets M2 internships, PhD theses and Post Doctoral studies. It will mainly take place at I2M laboratory under the overall supervision of Simon Essongue. Strong collaborations with both experts in AM physical aspects (Eric Lacoste at I2M laboratory) and ROM experts (the MONHADE Team at Inria Bordeaux and Tommaso Taddei at Sapienza Rome University) are to be expected.

We do not expect M2 internship applicants to have a state-of-the-art knowledge of nonlinear ROMs for Domain Decomposition but we will only consider those with a strong background in numerical methods for solving PDEs and the willingness to pursue a PhD.

Starting date:	M2 internship:	February 2027
	PhD student:	October 2027
	Post Doctoral Student:	February 2027

Mail to: simon.essongue-boussougou@u-bordeaux.fr

Please attach the following documents to your application:

- 1 cover letter
- 1 CV
- 1 transcript of grades (M1 and M2, if available; otherwise, please indicate at least the courses taken in M1 and M2)
- 1 internship or research report, if available/possible
- 1 list of publications, if available/possible
- 1 letter of recommendation

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