Introduction to the Hybrid High-Order method and implementation via DSEL

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SPEGEO school (2026)

NEMESIS
New generation methods for numerical simulations

Funded by the European Union

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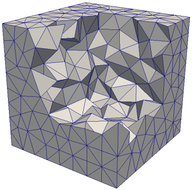
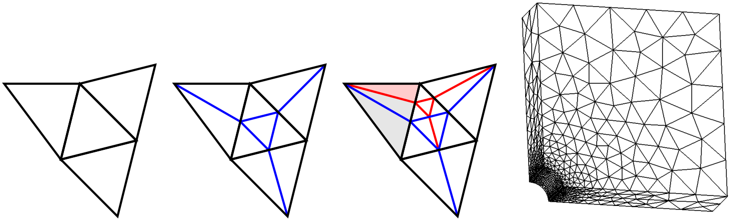


$\mathcal{T}_h = \{T\}$ conforming tetrahedral/hexahedral mesh.

- Define local polynomial spaces on each element, and glue them together to form a subspace of the continuous space associated with the model.
  Example: conforming $\mathcal{P}^1$ spaces in $H^1(\Omega)$.

- Gluing only works on special meshes...

The Finite Element way
Shortcomings



- Approach limited to conforming meshes with standard elements

$\Longrightarrow$ local refinement requires to trade mesh size for mesh quality

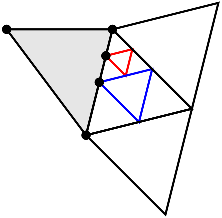
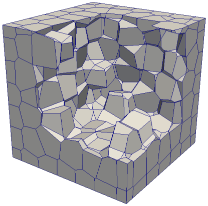
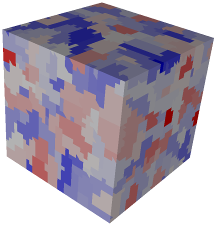
$\Longrightarrow$ complex geometries may require a large number of elements

$\Longrightarrow$ the element shape cannot be seamlessly adapted to the solution (e.g. hexahedra in boundary layers + tetrahedra in the bulk for CFD simulations)

- Need for (global) basis functions

$\Longrightarrow$ significant increase of DOFs on hexahedral elements

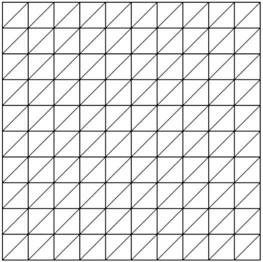
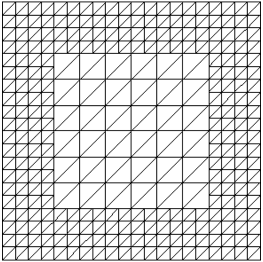
Benefits of polytopal meshes

- Local refinement is easy, and preserves mesh regularity.
- Agglomeration of elements (e.g., for multigrid methods) is seamless.
- High-level approach can lead to leaner methods (fewer DOFs).
- Can be combined with standard Finite Elements on hybrid meshes (made of tetrahedra/hexahedra + polyhedral elements).

Example of efficiency (I): Reissner–Mindlin plate problem

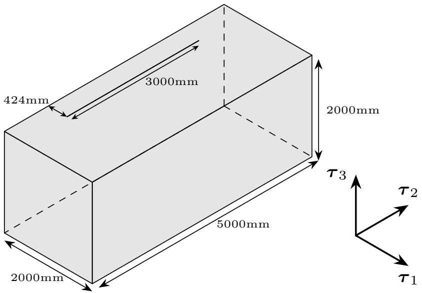
[Di Pietro and Droniou, 2022]

Stabilised $\mathcal{P}_2$-($\mathcal{P}_1 + \mathcal{B}^3$) scheme		DDR (polytopal) scheme	
			
nb. DOFs	Error	nb. DOFs	Error
2403	0.138	550	0.161
9603	6.82e-2	2121	6.77e-2
38402	3.40e-2	8329	3.1e-2

Example of efficiency (II): from CEA-CESTA

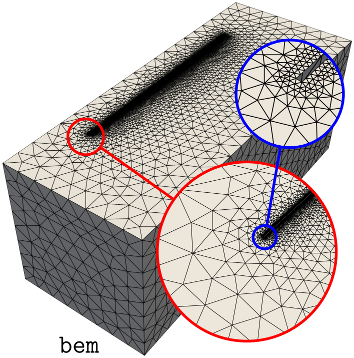
[Touzalin, 2025]

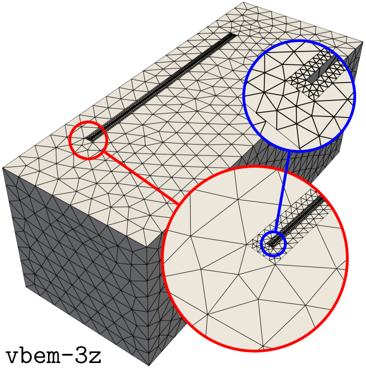
Problem: use a boundary element method to analyse the shielding effectiveness of a perfectly conductive box with a very small slit.



Example of efficiency (II): from CEA-CESTA

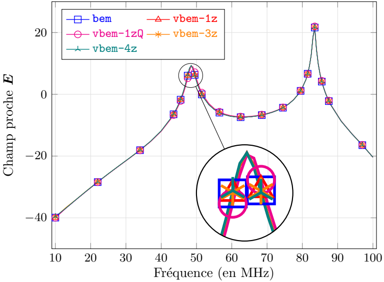
Meshes: conforming triangular for finite-element boundary method (bem), non-conforming triangular (polygonal) for virtual element boundary method (vbem-3z).

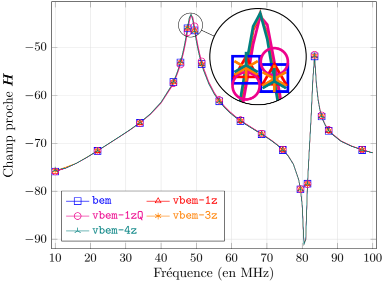




Example of efficiency (II): from CEA-CESTA

Accuracy: comparison of modulus of reflected near fields at the top.





Computational cost

Method	*Assembly*	*Resolution*
bem	813s	125s
vbem-3z	321s	19s

References

Di Pietro, D. A. and Droniou, J. (2022).
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